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14. ABSTRACT
The research supported by this grant consisted of developing improved algorithms and theory for designing and analyzing robust control systems for spatially interconnected systems. There are many examples of such systems, including automated highway systems, airplane formation flight, satellite constellations, cross-directional control in paper processing applications, and micro-cantilever array control for massively parallel data storage. One can also consider lumped approximations of partial differential equations - examples include the deflection of beams, plates, and membranes, and the temperature distribution of thermally conductive materials. Spatially interconnected control systems were studied in the context of fractional transformations on temporal and spatial shift operators, leading to multidimensional system optimization. These techniques were tested in simulation, and on the Cornell Formation Flight test-bed, in order to assess the validity of these approaches and to motivate further research.

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ROBUST AND OPTIMAL CONTROL OF SPATIALLY INTERCONNECTED SYSTEMS, WITH APPLICATION TO COORDINATED VEHICLE CONTROL

F49620-01-1-0119

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Abstract
The research supported by this grant consisted of developing improved algorithms and theory for designing and analyzing robust control systems for spatially interconnected systems. There are many examples of such systems, including automated highway systems, airplane formation flight, satellite constellations, cross-directional control in paper processing applications, and micro-cantilever array control for massively parallel data storage. One can also consider lumped approximations of partial differential equations – examples include the deflection of beams, plates, and membranes, and the temperature distribution of thermally conductive materials. Spatially interconnected control systems were studied in the context of fractional transformations on temporal and spatial shift operators, leading to multidimensional system optimization. These techniques were tested in simulation, and on the Cornell Formation Flight test-bed, in order to assess the validity of these approaches and to motivate further research.

Systems of Interest
The systems of interest that we considered in our research are captured in Figure 1.

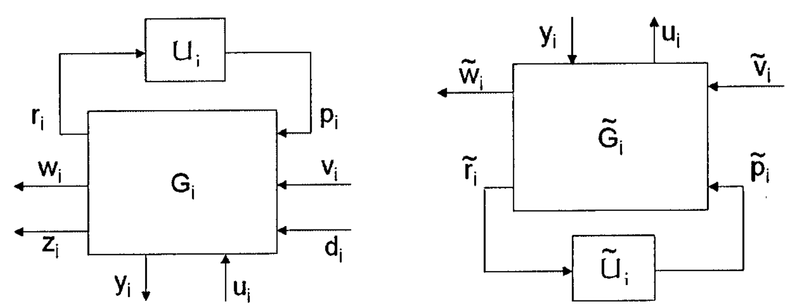

Figure 1: LEFT: Representative subsystem. RIGHT: Representative control subsystem.

Consider a collection of N dynamical systems:

$$((\dot{x}_i)(t), r_i(t), w_i(t), z_i(t), y_i(t)) = F_i\left(x_i(t), p_i(t), v_i(t), d_i(t), u_i(t)\right), \quad 1 \leq i \leq N, \quad (1)$$

where a representative subsystem is captured in Figure 1, and

1. Each F_i is a memoryless, nonlinear function, which belongs to some given set $\mathcal{F}_i$ (which could contain only one element). Set $\mathcal{F}_i$ can be used to capture parametric system uncertainty. Each F_i generates a state space system G_i, as per Figure 1.

2. Signals r_i and p_i are used to capture dynamic system uncertainty, and are required to be in some given set $\mathcal{U}_i$. For simplicity, this is loosely represented by the input-output block in Figure 1; in general, set $\mathcal{U}_i$ does not have to be defined as pairs of inputs and outputs.

3. Signals w_i and v_i are the interconnection variables. In particular,

$$w_i = (w_{i,1}, \cdots, w_{i,N}), \quad v_i = (v_{i,1}, \cdots, v_{i,N}), \quad w_{i,j} = v_{j,i}, \quad 1 \leq i \leq N, \quad 1 \leq j \leq N$$

$w_{i,j}$ is the signal originating from system i and terminating at system j, and $v_{i,j}$ is the signal terminating at system i and originating from system j.

4. Signals z_i capture the performance requirements, which consists of making z_i small in some given metric. Signals d_i capture the exogenous subsystem inputs, and consist of disturbances, reference signals, etc.

5. Signals y_i are the local sensor outputs, while signals u_i are the local actuator inputs.

Ultimately, we are interested in control design, and consider interconnections of the system described by (1) with what can be considered the control system, depicted in Figure 1:

$$\left(\dot{\tilde{x}}_i)(t), \tilde{r}_i(t), \tilde{w}_i(t), u_i(t)\right) = \tilde{F}_i(t)\left(\tilde{x}_i(t), \tilde{p}_i(t), \tilde{v}_i(t), y_i(t)\right), \quad 1 \leq i \leq N. \tag{2}$$

To place the class of systems in context, when $N = 1$, F is a fixed linear function and there are no signals p and r (no uncertainty), and no interconnection signals v and w, one could pose various induced gain control problems, such as H-infinity control, H-2 control, and L1 control. One can also consider various extensions and combinations of these problems, such as mixed performance objective optimization. With an appropriate choice of sets $\mathcal{F}$, $\mathcal{U}$, and signals r and p, this special case can be augmented to capture various types of system uncertainty. Many important results in this area were established in the last two decades, and this area of research is considered mature.

The research that we have been conducting as part of this grant has been focused on systems where N is large and the functions F_i are linear. The tools we have developed have been tried out on the Cornell Formation Flight Test-bed.

Summary of Results

Homogeneous, linear time invariant systems, without uncertainty are a special case of the above problem class: the F_i are linear and identical, there is only one element

in set $\mathcal{F}$, and there are no signals p_i and r_i. We have approached this problem class using semidefinite programming techniques. The relevant results are found in [1], [2], [3], [4], [5], [6], [7], [8], [9], [10], [11]. In these papers, we considered control design problems where each subsystem is interconnected to its nearest neighbors and form a lattice. This is depicted in Figure 2 for a one dimensional and a two dimensional periodic lattice.

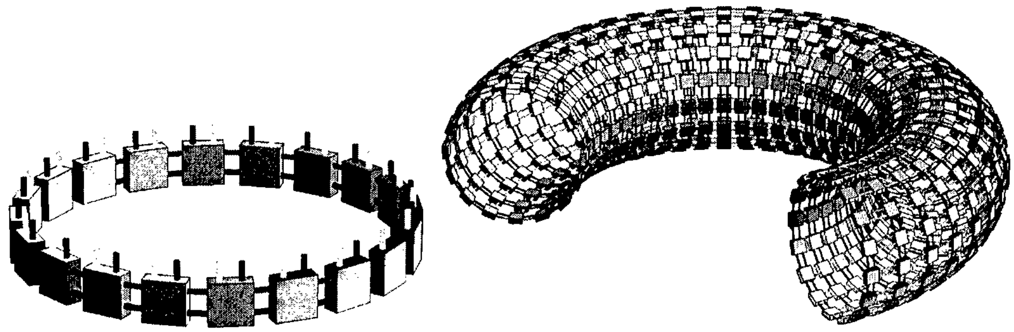

Figure 2: Left: one dimensional periodic lattice. Right: section of a two dimensional periodic lattice, with inputs and outputs omitted for clarity.

The optimization objective is to reduce the L-2 induced gain of the system. In particular, the following L-2 induced gain problem for continuous time systems can be posed. Consider the following N interconnected subsystems

$$(\dot{x}_i, w_i, z_i, y_i) = F(x_i, v_i, d_i, u_i), \quad w_{i,j} = v_{j,i}. \tag{3}$$

Design the following N interconnected subsystems (the control system)

$$\left(\dot{\tilde{x}}_i, \tilde{w}_i, u_i\right) = \tilde{F}(\tilde{x}_i, \tilde{v}_i, y_i), \quad \tilde{w}_{i,j} = \tilde{v}_{j,i} \tag{4}$$

such that the closed loop system

$$\left(\dot{\overline{x}}_i, \overline{w}_i, z_i\right) = \overline{F}(\overline{x}_i, \overline{v}_i, d_i), \quad \overline{w}_{i,j} = \overline{v}_{j,i} \tag{5}$$

is internally exponentially stable, and the following inequality is satisfied for all non-zero, square integrable d_i:

$$\int_{t=0}^{\infty} \sum_{i=1}^{N} |z_i(t)|_2^2 dt < \int_{t=0}^{\infty} \sum_{i=1}^{N} |d_i(t)|_2^2 dt \tag{6}$$

where $|\cdot|$ is the euclidian vector norm. This is depicted in Figure 3 for the case where the subsystems form a one dimensional periodic lattice. The resulting control synthesis conditions take the form of a semidefinite program:

$$\text{minimize } c^T q, \text{ subject to} \sum_{k=1}^{M} q_k A_k + A_0 < 0, \tag{7}$$

where vector c and symmetric matrices A_k are given, the q_i are the decision variables (the free parameters), and the inequality is understood in the sense of a quadratic form ($A < 0$ is equivalent to the eigenvalues of A being negative). The key features of this approach are the following:

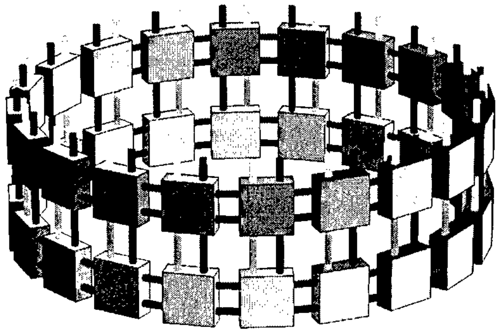

Figure 3: Closed loop system, one dimensional periodic lattice. The top lattice is the open loop plant, the bottom lattice is the control system.

1. The size of the resulting semidefinite program is only a function of the size of each subsystem. In particular, the number of decision variables M and the matrix dimensions of the A_k *do not grow with the number of interconnected elements* N.

2. The implementation of the control system is distributed. Each controller subsystem contains a computational unit, which is interconnected with its nearest neighbors. Global performance objectives can be obtained because the control system is connected via a distributed communication network.

3. The control subsystems do not need to be altered if the number of elements change, provided that the spatially invariant structure is preserved. The system can thus be reconfigured without being redesigned.

These tools have recently been used to design distributed control systems for a model of a vibrating cable [3], a vibro-acoustic problem [12], and a model of an adaptive secondary mirror for a high performance telescope [13], [14]. They have also been used to design distributed control strategies for the Cornell Formation Flight Test-Bed [15], [16], depicted in Figure 4. We have recently extended these results to the heterogeneous case, and to more general interconnection structures. Preliminary results may be found in [17], [18], [19], [20].

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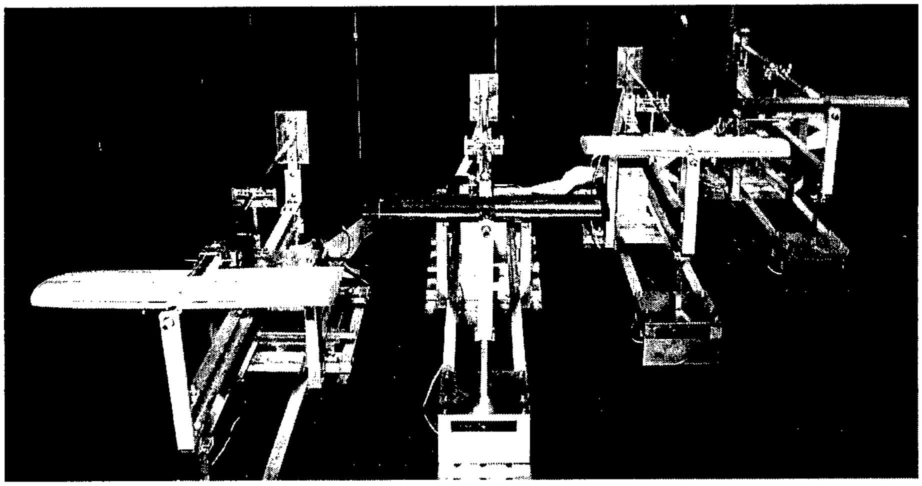

Figure 4: Cornell Formation Flight Test-Bed.

Personnel Supported During Duration of Grant

1. Raffaello D'Andrea (PI), Associate Professor, Cornell University.
2. JinWoo Lee, Post-doctoral research associate, Cornell University.
3. Ramu Chandra, Graduate Student, Cornell University.
4. Jeff Fowler, Graduate Student, Cornell University.

Publications

[1] R. D'Andrea and G. E. Dullerud. Distributed control design for spatially interconnected systems. *IEEE Transactions on Automatic Control*, 48(9):1478–1495, 2003.

[2] R. D'Andrea. A linear matrix inequality approach to decentralized control of distributed parameter systems. In *Proc. American Control Conference*, pages 1350–1354, 1998.

[3] R. D'Andrea. Linear matrix inequalities, multidimensional system optimization, and control of spatially distributed systems: An example. In *Proc. American Control Conference*, pages 2713–2717, 1999.

[4] R. S. Chandra, J. Fowler, and R. D'Andrea. Control of interconnected systems of finite spatial extent. *IEEE Conference on Decision and Control*, pages 238 – 239, 2002.

[5] C. Langbort and R. D'Andrea. Imposing boundary conditions for a class of spatially interconnected systems. In *Proc. American Control Conference*, pages 107–112, 2003.

[6] R. D'Andrea, C. Langbort, and R. Chandra. A state space approach to control of interconnected systems. In J. Rosenthal and D. S. Gilliam, editors, *Mathematical*

Systems Theory in Biology, Communication, Computation and Finance, pages 157–182. Springer, IMA Book Series, 2003.

[7] R. D'Andrea and R. S. Chandra. Control of discrete time interconnected systems. *IEEE Conference on Decision and Control*, pages 240–245, 2002.

[8] R. D'Andrea. Extension of Parrott's theorem to non-definite scalings. *IEEE Transactions on Automatic Control*, 45(5):937–940, 2000.

[9] C. Beck and R. D'Andrea. Minimality, controllability, and observability for a class of multi-dimensional systems. *IEEE Transactions on Automatic Control*, 2004. To appear.

[10] R. Chandra and R. D'Andrea. Necessity of the small gain theorem for multidimensional systems. In *Proc. IEEE Conference on Decision and Control*, pages 2859–2864, 2003.

[11] R. Chandra and R. D'Andrea. A scaled small gain theorem with applications to spatially interconnected systems. *IEEE Transactions on Automatic Control.* Submitted for publication.

[12] E. Scholte and R. D'Andrea. Active vibro-acoustic control of a flexible beam using distributed control. In *Proc. American Control Conference*, pages 2640 – 2645, 2003.

[13] B. Brandl, S. Eikenberry, T. Herter, and J. Kulkarni. An infrared optimized AO system for a 15-20m class telescope. In *SPIE Astronomical Telescopes and Instrumentation Conference*, 2002.

[14] J. Kulkarni, R. D'Andrea, and B. Brandl. Application of distributed control techniques to the adaptive secondary mirror of cornell's large atacama telescope. In *SPIE Astronomical Telescopes and Instrumentation Conference*, 2002.

[15] J. M. Fowler and R. D'Andrea. Distributed control of close formation flight. In *Proc. IEEE Conference on Decision and Control*, pages 2972–2977, 2002.

[16] J. M. Fowler and R. D'Andrea. A formation flight experiment: Constructing a test-bed for research in control of interconnected systems. *Control Systems Magazine*, 23(5):35–43, 2003.

[17] C. Langbort and R. D'Andrea. Distributed control of heterogeneous systems interconnected over arbitrary graphs. In *Conference on Decision and Control*, pages 2835–2840, 2003.

[18] C. Langbort, R. Chandra, and R. D'Andrea. Distributed control of heterogeneous systems interconnected over an arbitrary graph. *IEEE Transactions on Automatic Control.* To appear.

[19] B. Recht and R. D'Andrea. Distributed control of systems over discrete groups. *IEEE Transactions on Automatic Control.* To appear.

[20] B. Recht and R. D'Andrea. Exploiting symmetry for the distributed control of spatially interconnected systems. In *Conference on Decision and Control*, pages 598–603, 2003.

Honors and Awards

1. RoboCup World Champions, F180 League, Padova, Italy, 2003. System architect and faculty advisor, Cornell Autonomous Robotic Soccer team.
2. Plenary Speaker, American Control Conference, 2003.
3. Graduate student Cedric Langbort finalist, best student paper award, ACC 2003.
4. Keynote Speaker, University of Toronto Engineering Science Annual Dinner, 2003.
5. Special Topic Invited Speaker, Mathematical Theory of Networks and Systems Conference, 2002.
6. RoboCup World Champions, F180 League, Fukuoka, Japan, 2002.
7. Presidential Early Career Award for Scientists and Engineers (PECASE), 2001.
8. Plenary Speaker, SIAM Conference on Control and its Applications, 2001.
9. RoboCup Third Place Winners, F180 League, Seattle, USA, 2001.
10. Distinguished Lecturer, National Science Foundation Research Highlight Series, 2001.

Transitions and Seminars

1. University of Pennsylvania, Mechanical Engineering and Applied Mechanics Department, November 2003. "Design of Robust Multi-Vehicle Systems."
2. Stanford University, Aerospace Engineering Department, September 2003. "Design of Robust Networked Control Systems ."
3. Charles River Analytics, Boston, MA, August 2003. "Design and Control of Robust Large Scale Systems."
4. Spoletoscienza, Spoleto Festival, Italy, July 2003. "Control of Autonomous and Semi-Autonomous Vehicles."
5. University of Padova, Italy, Dipartimento di Ingegneria dell'Informazione, July 2003. "Design of Robust Networked Control Systems."
6. University of Illinois at Urbana, Aeronautical and Astronautical Engineering Department, April 2003. "Cooperative Vehicle Control."
7. Vanderbilt Electrical Engineering and Computer Science Lecture Series, Nashville, TN, November 2002. "Control of Complex Systems."
8. The Center for Bits and Atoms, Massachusetts Institute of Technology, Boston, MA, October 2002. "Control of Complex Systems."
9. University of Florida, Graduate Engineering Research Center, Research Institute for Autonomous Precision Guided Systems, Shalimar, Florida, May 2002. "Cooperative Vehicle Control."
10. GRASP Laboratory, University of Pennsylvania, Philadelphia, Pennsylvania, May 2002. "Cooperative Vehicle Control."
11. AFOSR Workshop on Future Directions in Control, Arlington, Virginia, April 2002. "Controlling Structured Spatially Interconnected Systems."

12. NASA Langley Research Center, Hampton, Virginia, June 2001. "Cooperative Control of Autonomous and Semi-Autonomous Vehicles." (Colloquium Lecture).
13. University of California, Santa Barbara, Mechanical and Environmental Engineering, January 2001. "Control of Complex Systems."